

Idiomatic specification of magic wands in implicit dynamic frames

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Implicit dynamic frames in a nutshell

Verifying heap-manipulating programs...

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method mergesort(root: Ref)
  requires list(root)
  ensures  list(root) && sorted(root) && ...
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`acc(x.val)`

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`acc(x.val)` `x.val` **heap-dependent-expression**

```
predicate list(x: Ref) { x ≠ null ⇒ acc(x.val) && acc(x.next) && list(x.next) }
```

```
function sorted(x: Ref): Bool
  requires list(x) {
    x ≠ null && x.next ≠ null ⇒ x.val ≤ x.next.val && sorted(x.next)
  }
```

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Magic wand $A \dashv\vdash B$ can be *applied* to exchange A for B : represent partial data structures (“ B but A is missing”)

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method borrow_fst(x: Ref)
  requires pair(x)
  ensures  acc(x.fst) && (acc(x.fst)  $\dashv\!\ast$  pair(x))
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Magic wand $A \multimap B$ can be *applied* to exchange A for B : represent partial data structures (“ B but A is missing”)

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Heap-dependent expressions

- framing $x.\text{fst}$: valid thanks to $\text{acc}(x.\text{fst})$

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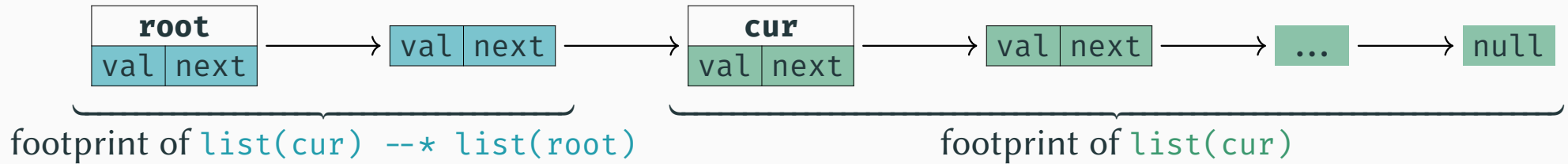
- framing $x.\text{fst}$: valid thanks to $\text{acc}(x.\text{fst})$
- framing $x.\text{snd}$: intuitively valid thanks to $\text{acc}(x.\text{fst}) \multimap \text{pair}(x)$... but not possible in existing verifiers

Magic wands in a nutshell (2)

Magic wand $A \dashv\dashv B$ can be *applied* to exchange A for B : represent partial data structures (“ B but A is missing”)

Typical use case #2

```
while (cur  $\neq$  null)
  invariant list(cur)
  invariant list(cur)  $\dashv\dashv$  list(root)
```

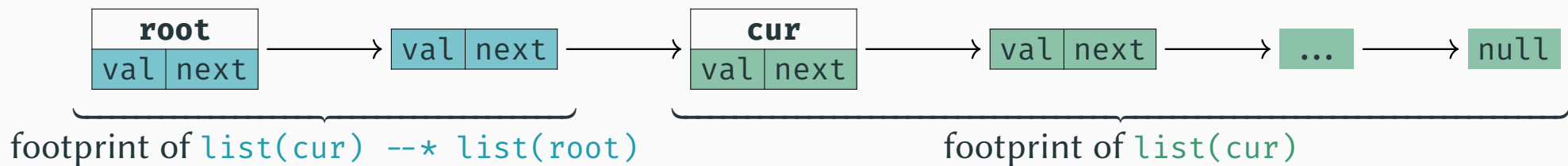


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Heap-dependent expressions: predicates vs wands

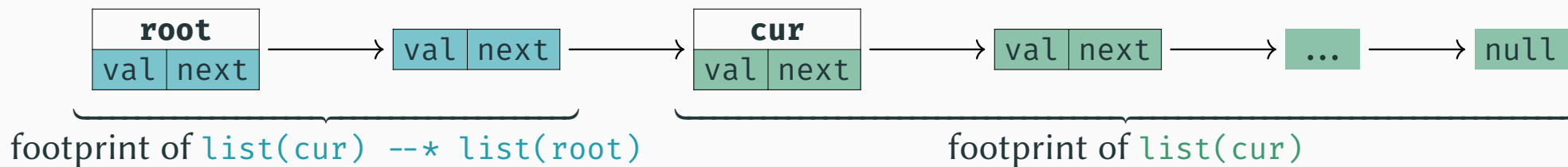
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Heap-dependent expressions: predicates vs wands

- **predicates**: framing cur.val : **unfolding** $\text{list}(\text{cur})$ **in** cur.val
- **magic wands**: framing root.val : **???** $\text{list}(\text{cur}) \multimap \text{list}(\text{root})$ **in** root.val

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Tasks

1. Design a new IDF construct
2. Implementation in the **automated** verifier Viper (symbolic execution backend)

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- discovered and fixed **soundness** issues
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- highly **automated** (come for free when packaging)
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Attached expressions

- design space exploration
- theoretical **sufficient validity conditions**

Main challenge: the footprint of a wand is not explicit (1)

Predicates (explicit footprint)

```
predicate list(x: Ref) {  
  x  $\neq$  null  $\implies$  acc(x.val) && acc(x.next) && list(x.next)  
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Wands (implicit footprint)

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Vacuous wands

Formally: $\sigma \models A \text{ --* } B \stackrel{\text{def}}{\iff} \forall \sigma_A : \sigma_A \perp \sigma \wedge \sigma_A \models A \Rightarrow \sigma_A \uplus \sigma \models B$

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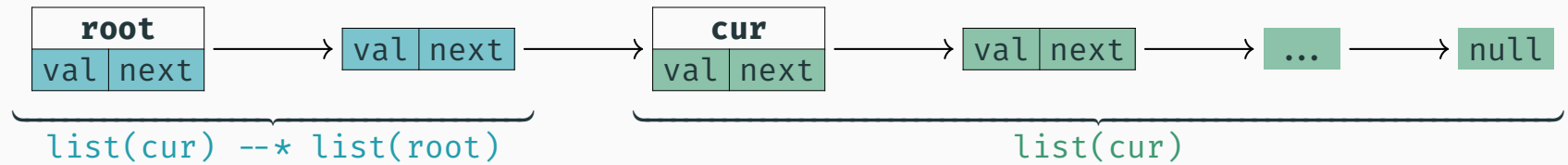
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- `false` --* anything
- `acc(x.fst)` --* `pair(x)`

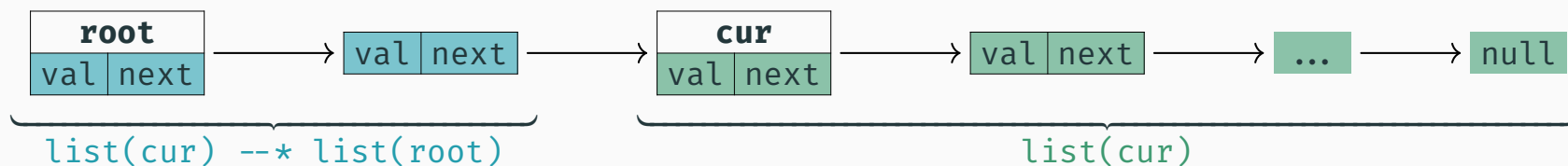
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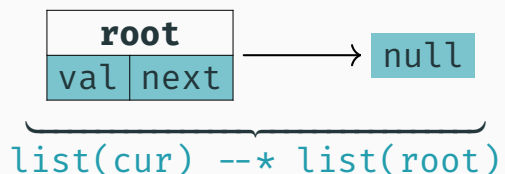


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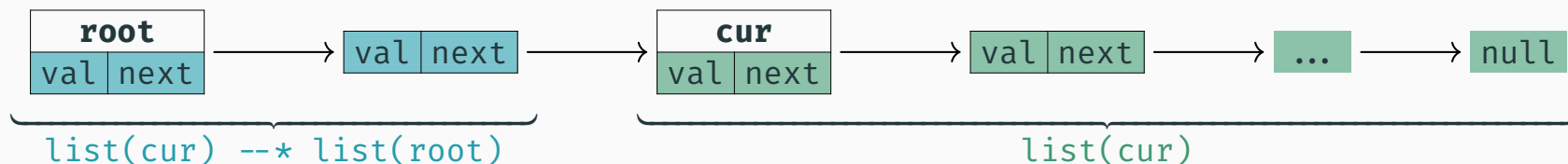
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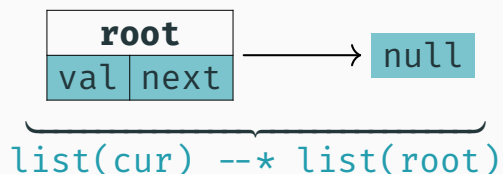
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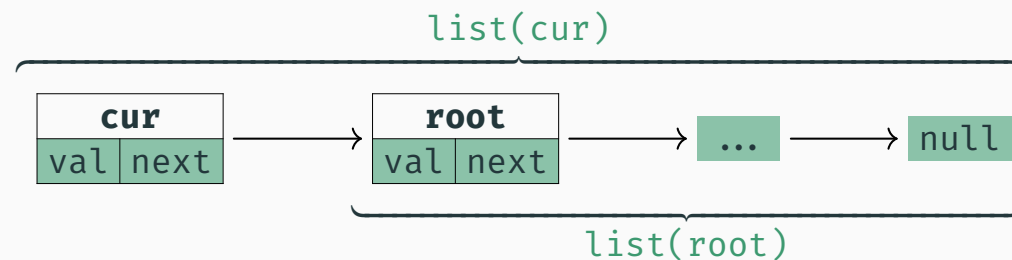
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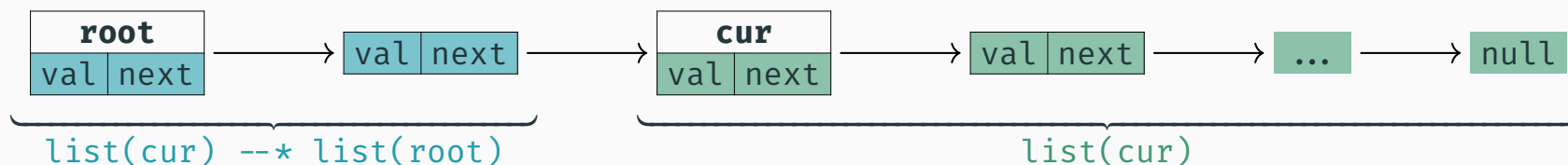
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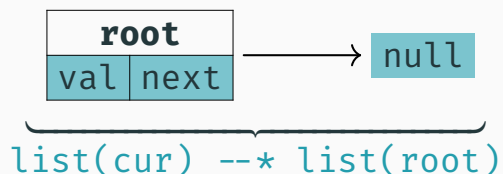
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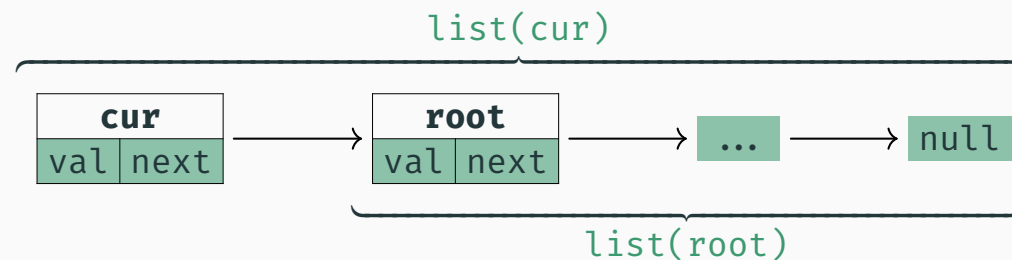
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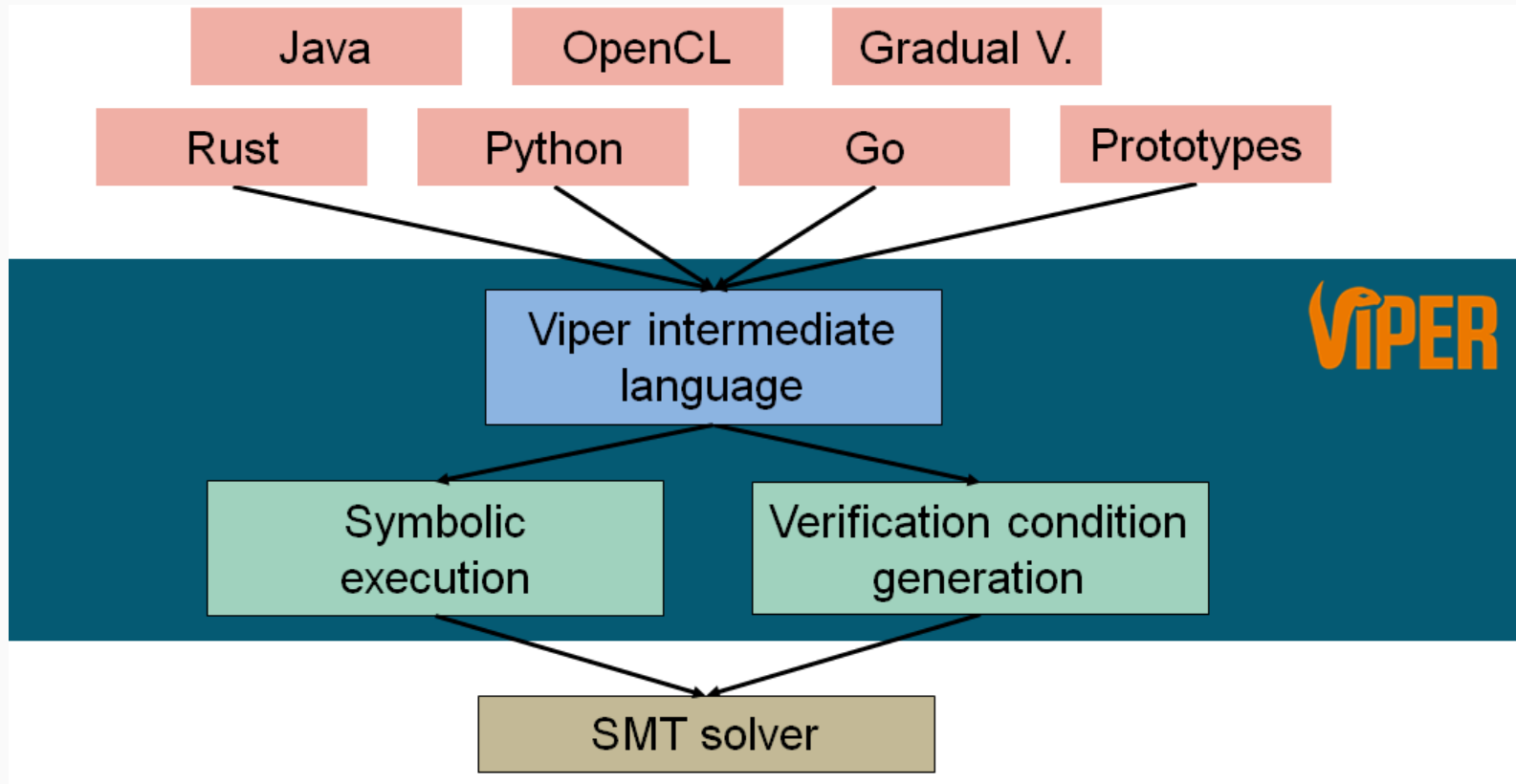
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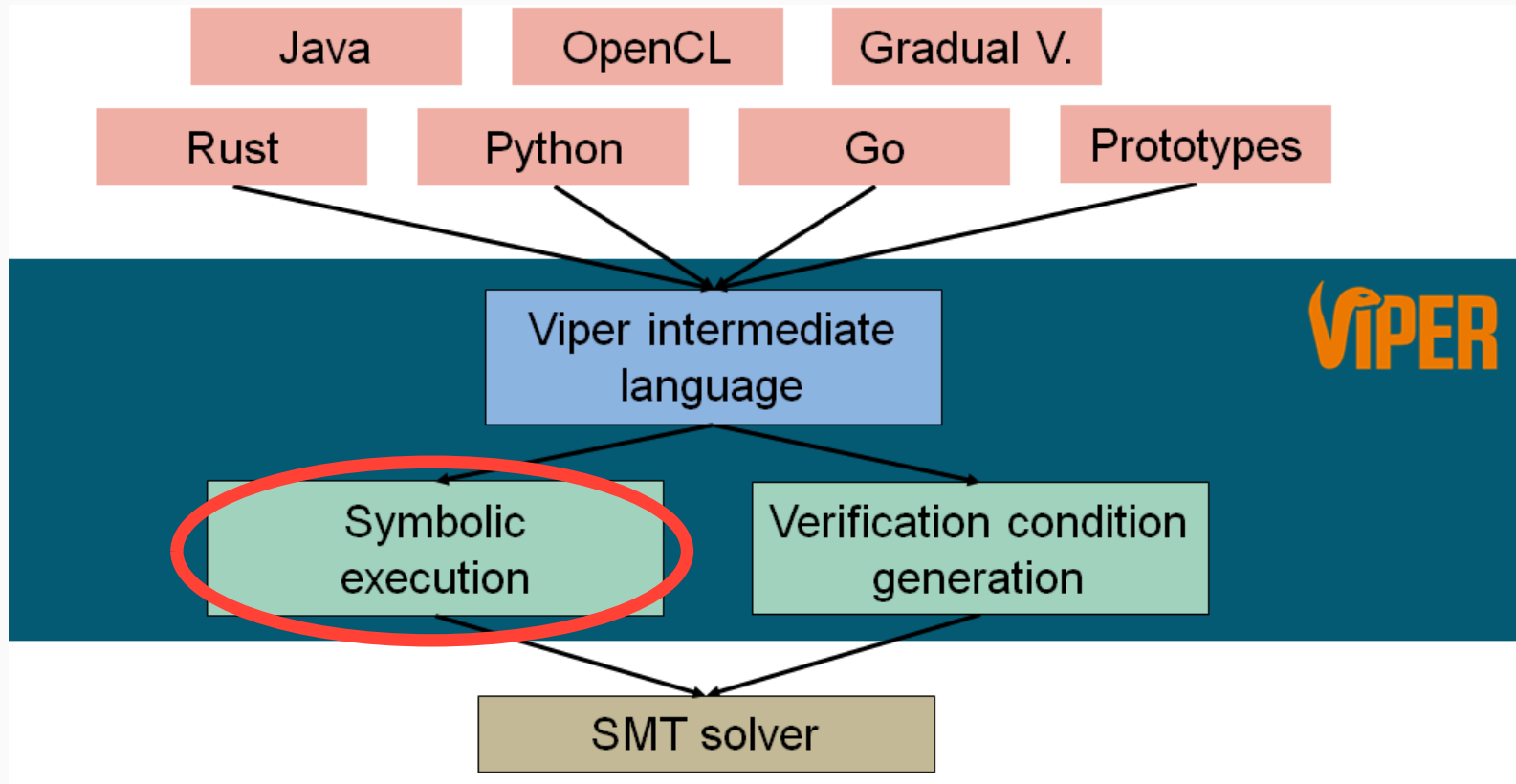
Conclusion: the wand's syntax is not enough to infer its footprint.

An overview of Viper



From viper.ethz.ch (accessed on 24.07.2026)

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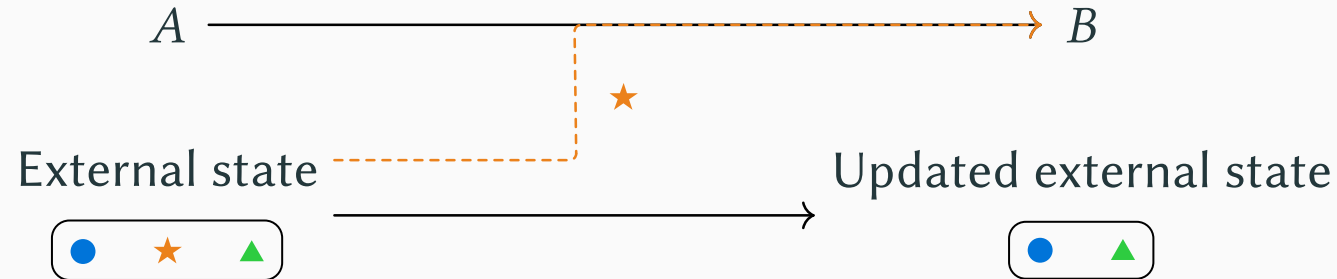
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Wands in Viper

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Exchanging A and the wand for B

Packaging a wand $A \dashv\dashv \star B$



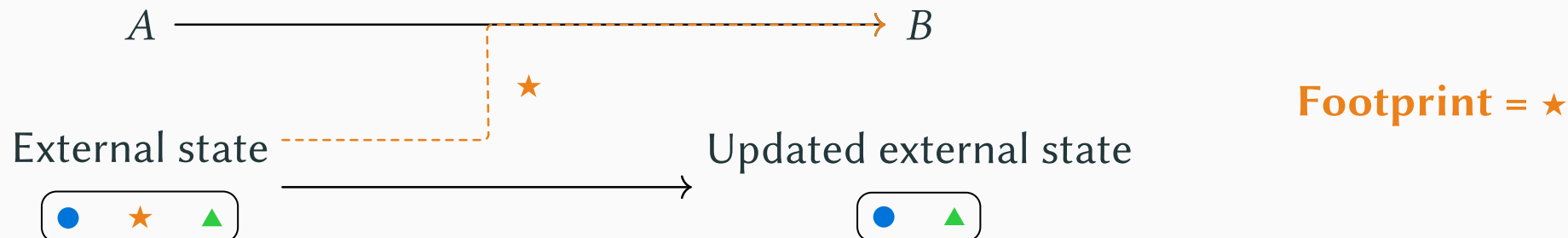
Footprint = $\star$

Wands in Viper

Applying a wand $A \multimap B$

Exchanging A and the wand for B

Packaging a wand $A \multimap B$



Why don't we just record the footprint?

- not all wands are packaged: what about wands from preconditions/invariants/...?
- would require significant modifications to Viper
- less expressive than the proposed solution

Modular functional specifications for wands

Attached facts by example (1)

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--* list(root)
&& len(root) = old(len(root))
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Wand and attached facts

```
list(cur) --* list(root)

attached old[lhs](len(cur)) = len_cur
    ⇒ len(root) = old(len(root))
to list(cur) --* list(root)

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- **incremental** specification: first memory safety using the wands
- **modular** functional spec: each functional property specified and verified independently

Attached facts by example (2)

Attached facts come for free

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while ( ... )  
  invariant w  
  invariant attached F to w  
  invariant attached G to W  
{  
  ...  
  package w { ... }  
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Weakening properties

```
method weaken(x: Ref)  
  requires acc(x.fst) --* pair(x) /* w */  
  requires attached unfolding pair(x) in x.snd  $\geq$  42 to w  
  ensures w  
  ensures attached unfolding pair(x) in x.snd  $\geq$  0 to w  
{ /* We don't need to do anything to prove the  
  postcondition. */ }
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Modular spec for wands

```
function f( ... ): ...  
  requires A && P1l --* B && P1r  
  
method m( ... ) {  
  package A && P1l && P2l --* B && P1r && P2r  
  f( ... ) // error: wand not found  
}
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```
function f( ... ): ...  
  requires A --* B  
  requires attached old[lhs](P1l)  $\implies$  P1r to A --* B
```

```
method m( ... ) {  
  package A --* B  
  f( ... ) // works fine  
}
```

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Context: `attached e to A --* B`

- `e` is a (pure) boolean expression (no `acc( ... )`, predicates, or wands)

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$$\llbracket \text{attached } e \text{ to } w \rrbracket \stackrel{\text{def}}{=} \forall t_A : \text{MW_token}(w, t_A) \Rightarrow \llbracket e \rrbracket$$

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 - when applying a wand with a concrete LHS t_{LHS} , yield a token $\text{MW_token}(w, t_{\text{LHS}})$
 - also used in the new encoding to fix the soundness issues

On examples

Successful rewrites of examples from papers [1], [2] + new ones (e.g. `delete_last.vpr`, `stalin-sort.vpr`)

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Early experiments with Prusti [5], [6]

Very promising on a few examples (e.g. binary search tree implem), wands naturally appear in Rust

```
struct Pair { fst: i32, snd: i32 }           predicate pair(x: Ref) { acc(x.fst) && acc(x.snd) }

fn borrow_first(p: &mut Pair) → &mut i32 { method borrow_first(p: Ref)
    &mut p.fst                               requires pair(p)
}                                             ensures acc(p.fst)
                                             ensures acc(p.fst) --* pair(p)
```

Accessing the footprint using attached expressions

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Examples

Accessing the second element of a pair:

```
x.snd := 0
package acc(x.fst) --* pair(x) { ... }
var v: Int := attachedexp unfolding pair(x) in x.snd to W
assert v = 0
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(Computed) length of the prefix list:

```
while ( ... )
  invariant list(cur) --* list(root)
  invariant ...
  invariant (attachedexp len(root) - old[lhs](len(cur)) to W) = i
{ ... }
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(Computed) length of the prefix list:

```
while ( ... )
  invariant list(cur) --* list(root)
  invariant attachedexp_valid len(root) - old[lhs](len(cur)) to W
  invariant      (attachedexp len(root) - old[lhs](len(cur)) to W) = i
{ ... }
```

Semantics

$\llbracket \text{attachedexp } e \text{ to } w \rrbracket := \llbracket e \rrbracket$ checking the conditions below

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Independence from the LHS

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$\forall t_1, t_2 : \text{MW_token}(w, t_1) \wedge \text{MW_token}(w, t_2) \Rightarrow \llbracket e \rrbracket[t_A := t_1] = \llbracket e \rrbracket[t_A := t_2]$

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- **not sufficient**: need a token to use that property: check the wand is not vacuously true

Appliability

Recall $\sigma \vDash A \multimap B \iff \forall \sigma_A : \sigma_A \perp \sigma \wedge \sigma_A \vDash A \Rightarrow \sigma_A \uplus \sigma \vDash B$

A wand is *applicable* if there exists a valid LHS: $\exists \sigma_A : \sigma_A \perp \sigma \wedge \sigma_A \vDash A$

Validity conditions and semantics of attached expressions

Semantics

$\llbracket \text{attachedexp } e \text{ to } w \rrbracket := \llbracket e \rrbracket$ checking the conditions below

Independence from the LHS

$\llbracket \text{attachedexp_valid } e \text{ to } w \rrbracket \stackrel{\text{def}}{=}$

$$\forall t_1, t_2 : \text{MW_token}(w, t_1) \wedge \text{MW_token}(w, t_2) \Rightarrow \llbracket e \rrbracket[t_A := t_1] = \llbracket e \rrbracket[t_A := t_2]$$

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Automation/integration in Silicon = future work

Conclusion

A sound algorithm and encoding for packaging path conditions

- introducing **magic wand tokens**: new sound encoding, same performance

Attached facts

- enable **incremental** and **modular** functional specification of magic wands
- highly **automated** (come for free when packaging)
- promising application to Rust in Prusti

Attached expressions

- design space exploration
- **sufficient validity conditions**  independence from the LHS  applicability

Future work

Application to Rust

Automation of applicability

Formalization of this work

All artifacts available on github [remigerme/silver](#) and [remigerme/silicon](#) (see branches)

A sound package algorithm for path conditions

Silicon: the symbolic execution backend of Viper

Symbolic state

Symbolic variables

Silicon: the symbolic execution backend of Viper

Symbolic state

Symbolic variables + symbolic heaps

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Symbolic variables + symbolic heaps + path conditions

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Internal procedures

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e must be well-framed

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 - `consume(x.f ≥ 0)`: regular assertion
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Silicon: the symbolic execution backend of Viper

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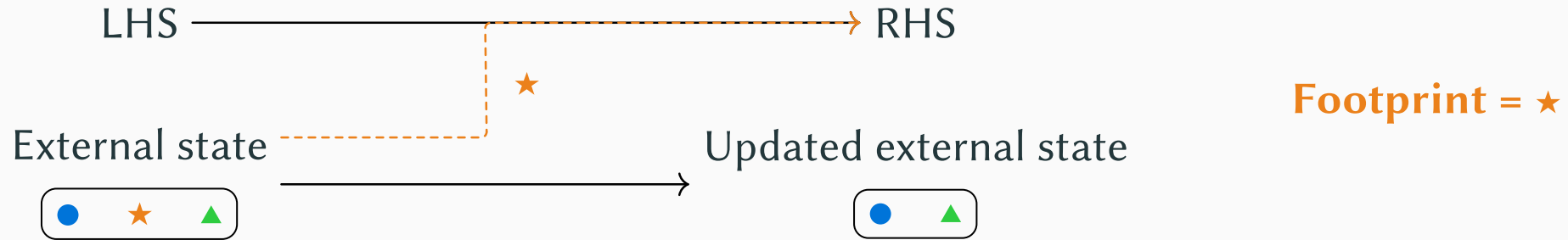
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- `produce(a: Assertion, t: Term, s: State)(Q: State => ...)`
adds the resources in a and assumes a holds

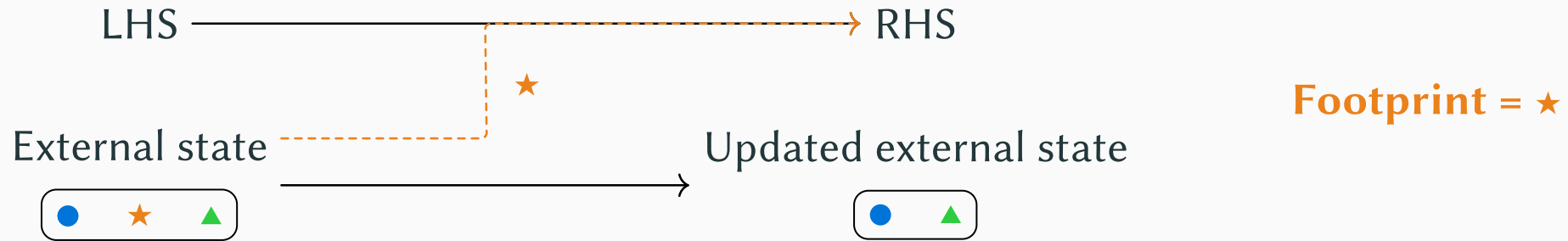
The existing package algorithm

Packaging a wand



The existing package algorithm

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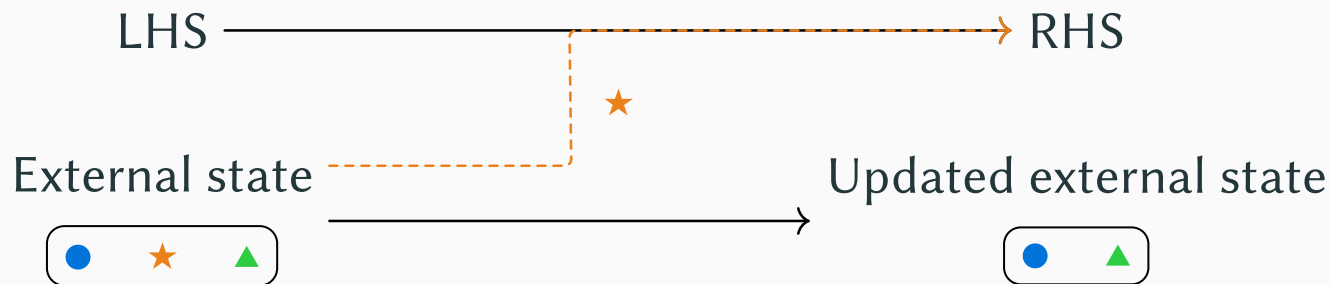


Overview of the package algorithm

1. produce LHS t_A
 2. execute proof script
 3. consume RHS
- Any of these steps might **branch** and put resources in the footprint
- For each recorded branch:

The existing package algorithm

Packaging a wand



Footprint = ★

Overview of the package algorithm

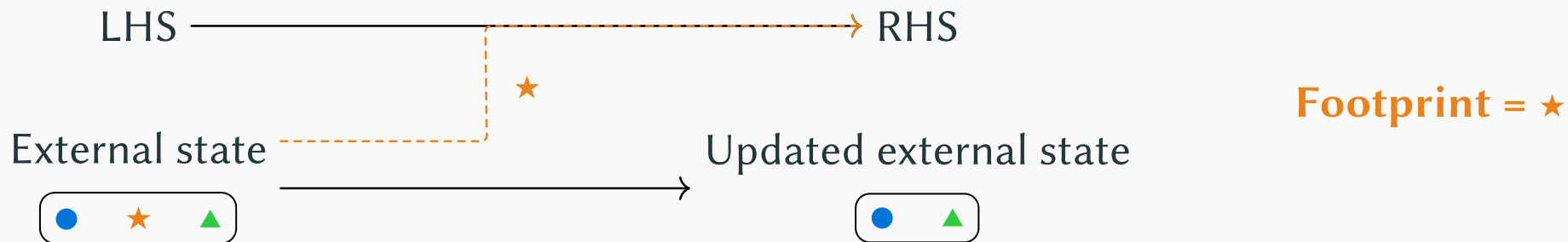
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5. handle path conditions (intricate)
6. call the continuation

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Intuition: unsound to assume the branch at package time

A program showcasing the unsoundness of the algorithm

```
acc(x.b) --* acc(x.b) && (x.b ? acc(x.f) : acc(x.g))
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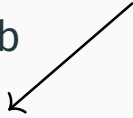


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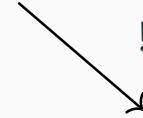
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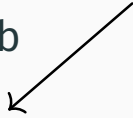


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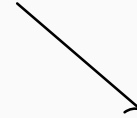


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`applying w in true`

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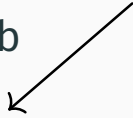


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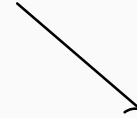


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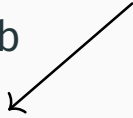


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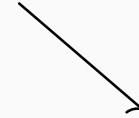
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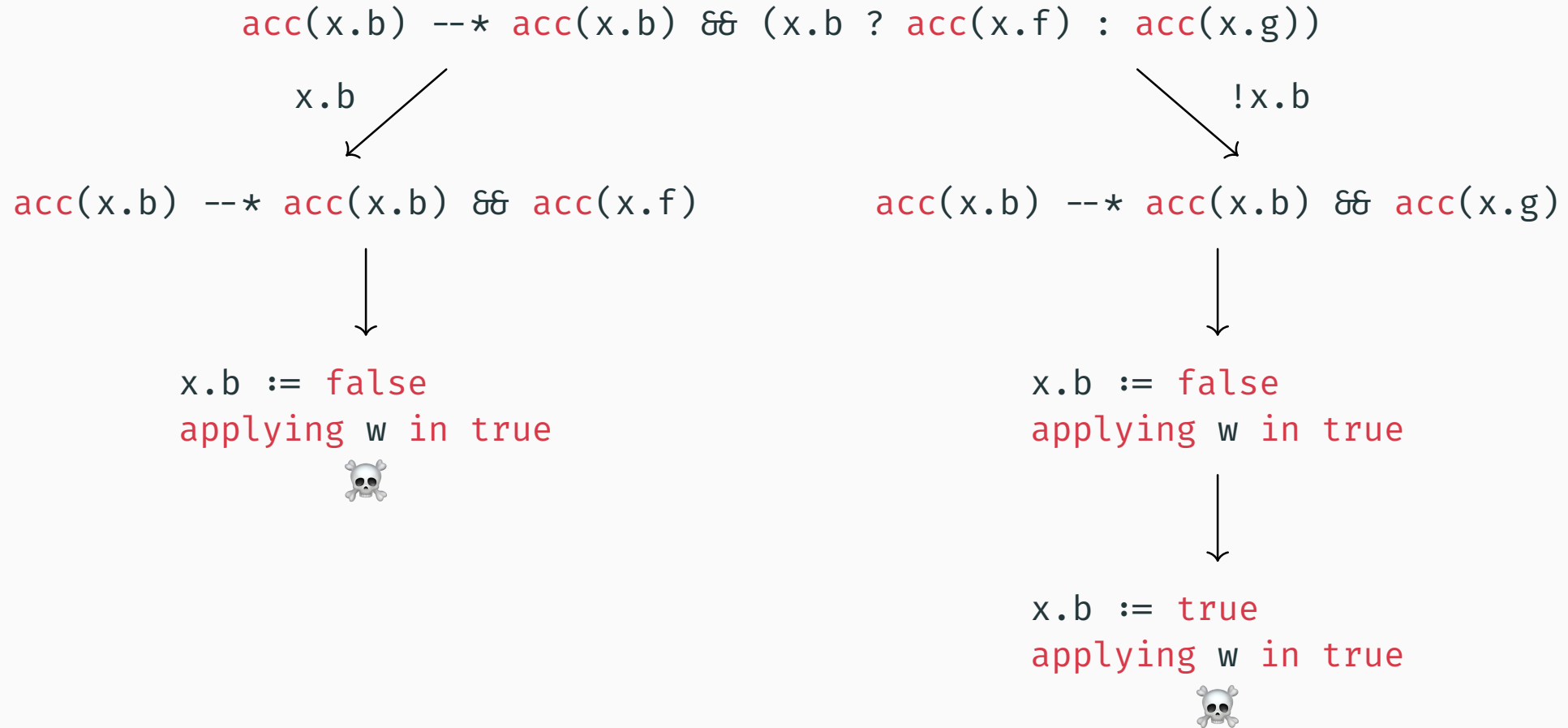


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A program showcasing the unsoundness of the algorithm



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Unsound: `assert false` succeeds in both branches!

A sound packaging algorithm

The fix

1. join all recorded branches (states and path conditions)
2. verify the rest of the program only once

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Yield a token $\text{MW_token}(w, t_{LHS})$ when applying w with concrete LHS t_{LHS}

Evaluation of the new encoding

Against the test suite

178 test files with wands (all passing with the previous encoding):

- 166 still verify
- 12 failing:

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Benchmark

On large verified codebases (Internet router [3] / verification library for security protocols [4])

Equivalent performance (details)

Example of joining heaps

Branch condition	Remaining heap	Conditionalized heap	Final joined heap
$x.b - \text{First}(\text{mwsf}(t_A))$	$(x.g \rightarrow \dots \# 1)$	$(x.g \rightarrow \dots \# x.b ? 1 : 0)$	$(x.g \rightarrow \dots \# x.b ? 1 : 0,$ $x.f \rightarrow \dots \# !x.b ? 1 : 0)$
$!x.b - !\text{First}(\text{mwsf}(t_A))$	$(x.f \rightarrow \dots \# 1)$	$(x.f \rightarrow \dots \# !x.b ? 1 : 0)$	

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$!x.b - !\text{First}(\text{mwsf}(t_A))$	$(x.f \rightarrow \dots \# 1)$	$(x.f \rightarrow \dots \# !x.b ? 1 : 0)$	

Wait, t_A is dangling... fine: this prevents the solver from accessing $x.f$ and $x.g$.

Details of the tests

Failing test file	Comment
conditionals.vpr	should have been failing in the first place as they exhibit a wrong footprint computation for wands (essentially described in [7])
SnapshotsBranching.vpr	
conditionals3.vpr	can be successfully verified by enabling the SE-PC flag
PackageStateConsolidator.vpr	
packaging_1.vpr	true completeness regression

conditionals.vpr and SnapshotsBranching.vpr

```
method branching(x: Ref)
  requires acc(x.f) && acc(x.g, write) &&
acc(x.h, write)
{
  define A acc(x.f)
  define B acc(x.f) && (x.f ? acc(x.g) :
acc(x.h))

  package A --* B
  apply A --* B

  assert acc(x.f) && acc(x.g) && acc(x.h)
}
```

packaging_1.vpr

```
method incomplete(x)
  requires acc(x.f, 1/2)
{
  package acc(x.f, 1/4) && x.f = 2 --* acc(x.f,
1/4) && x.f = 2 {
    package true --* acc(x.f, 1/2) && x.f = 2
  }
}
```

Go back

Benchmarking the new encoding

Benchmarking on [3], [4] using hyperfine [8]¹:

Encoding	Mean [s]	Min [s]	Max [s]	Relative
old	139.253 ± 21.251	123.357	173.944	1.05 ± 0.17
new	132.219 ± 7.593	124.566	143.658	1.00

Table 1: Hyperfine benchmark results for the Internet router (5 runs per encoding).

Encoding	Mean [s]	Min [s]	Max [s]	Relative
old	25.433 ± 0.471	24.367	26.227	1.00
new	26.129 ± 0.647	25.138	27.397	1.03 ± 0.03

Table 2: Hyperfine benchmark results for the security protocol verification library (15 runs per encoding).

Go back

¹All source files and the benchmark script can be found on my repository <https://github.com/remigerme/silicon/blob/9ca9b342a9ad12dc888b573d7ca046ba9f749783/remi-demo>

Generic evaluation procedure

Context: **attached** e **to** $A \dashv\dashv B$. Setup to evaluate e as in $A \dashv\dashv B$ && e :

1. produce artificial LHS t_A
2. produce artificial RHS $mwsf(t_A)$
3. eval e in the resulting state: **the resulting term t_e depends on t_A**

```
def evalAttachedGeneric(e: Exp, w: Wand, s: State,
                        evalExp: (State, MWSF, Var, Term) => Term,
                        Q: (State, Term) => ...) => ...:
  consume(w, s)((mws, sW) =>
    tA = freshTerm()
    produce(w.lhs, tA, sW)(sA =>
      produce(w.rhs, mws.apply(tA), sA)(sB =>
        eval(e, sB)(tE =>
          tRes = evalExp(sB, mws, tA, tE)
          Q(sE, tRes)
        )
      )
    )
  )
```

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