Idiomatic specification of magic wands in implicit dynamic frames

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Implicit dynamic frames in a nutshell

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IDF: permission reasoning $\oplus$ functional specification

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```
predicate list(x: Ref) { x ≠ null ⟹ acc(x.val) && acc(x.next) && list(x.next) }

function sorted(x: Ref): Bool
  requires list(x) {
  x ≠ null && x.next ≠ null ⟹ x.val ⩽ x.next.val && sorted(x.next)
}
```

Magic wands in a nutshell

Magic wand A `--*` B can be *applied* to exchange A for B

Typical use case

```
while(cur ≠ null)
  invariant list(cur)
  invariant list(cur) --* list(root)
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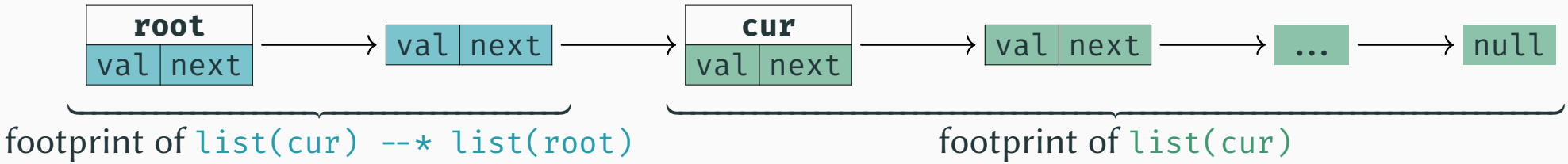

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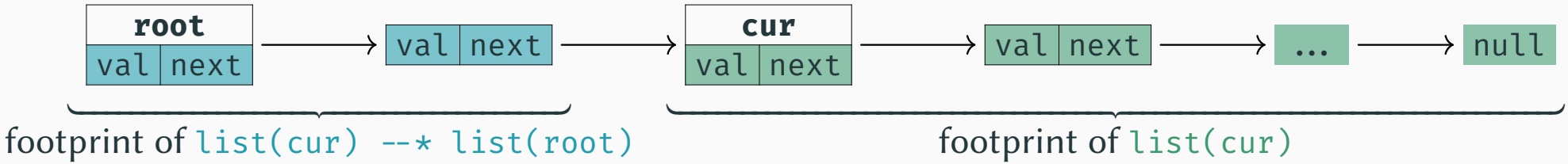

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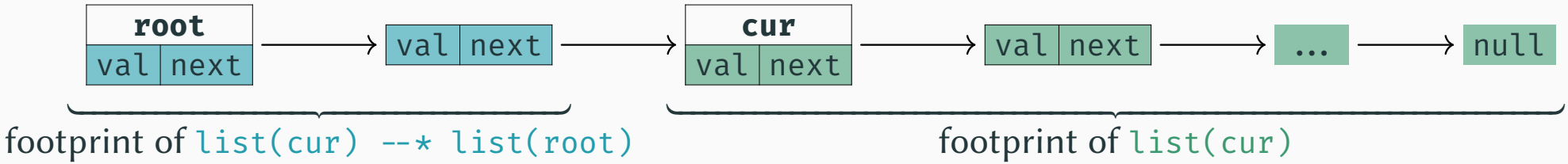

Heap-dependent expressions: predicates vs wands

- **predicates:** framing `cur.val`: `unfolding list(cur) in cur.val`
- **magic wands:** framing `root.val` `??? list(cur) --* list(root) in root.val`

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Tasks

1. Design a new IDF construct
2. Implementation in the **automated** verifier Viper (symbolic execution backend)

Contributions

A sound algorithm and encoding for packaging path conditions

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- enable **incremental** and **modular** functional specification of magic wands
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Attached expressions

- design space exploration
- theoretical **sufficient validity conditions**

Main challenge: the footprint of a wand is not explicit (1)

Predicates (explicit footprint)

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Formally: $\sigma \vDash A \mathrel{-\!\!*} B \overset{\text{def}}{\iff} \forall \sigma_A : \sigma_A \perp \sigma \wedge \sigma_A \vDash A \Rightarrow \sigma_A \uplus \sigma \vDash B$

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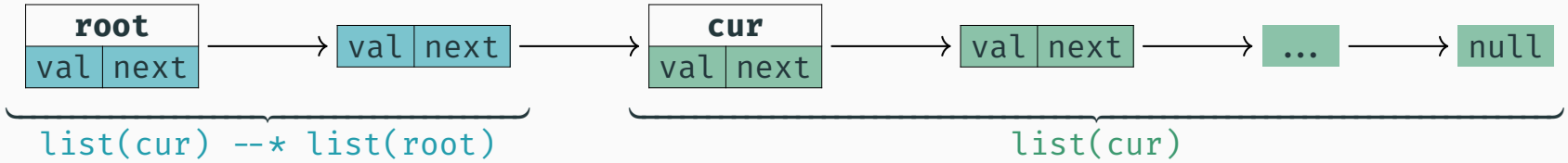

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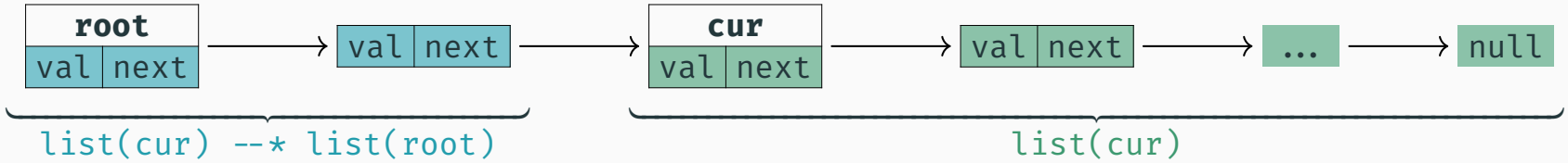

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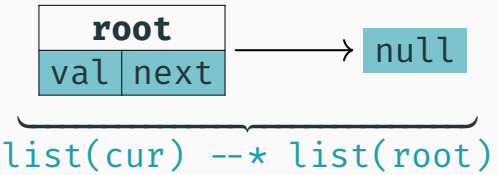

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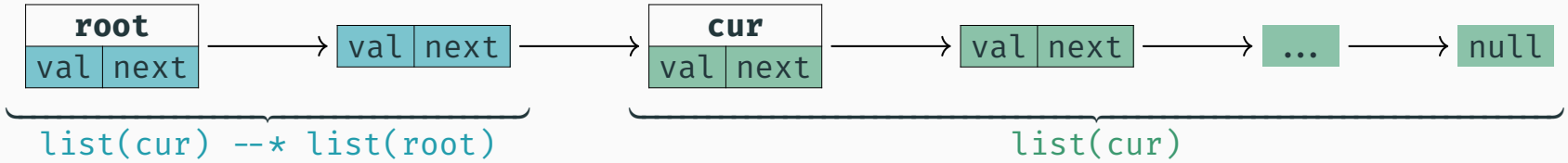

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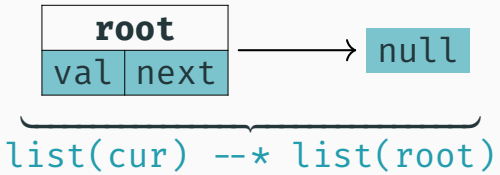

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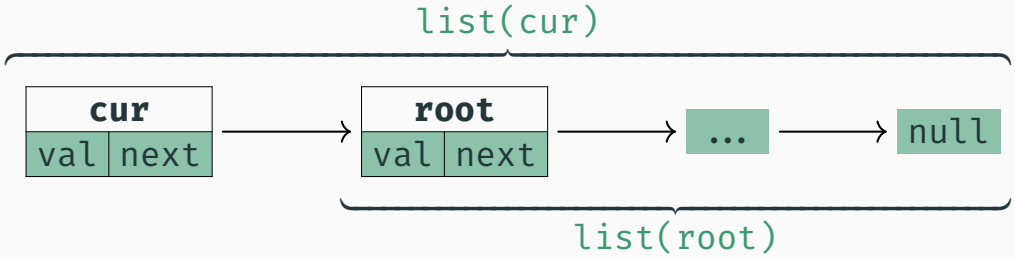

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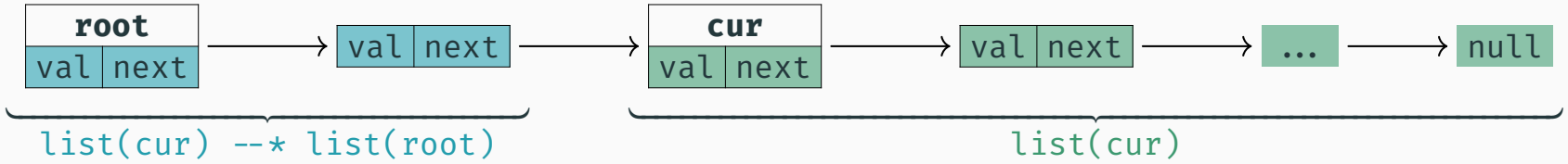

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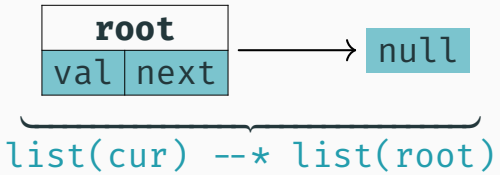

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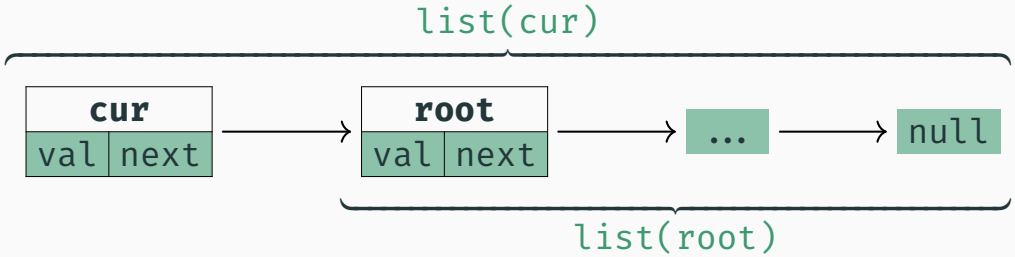
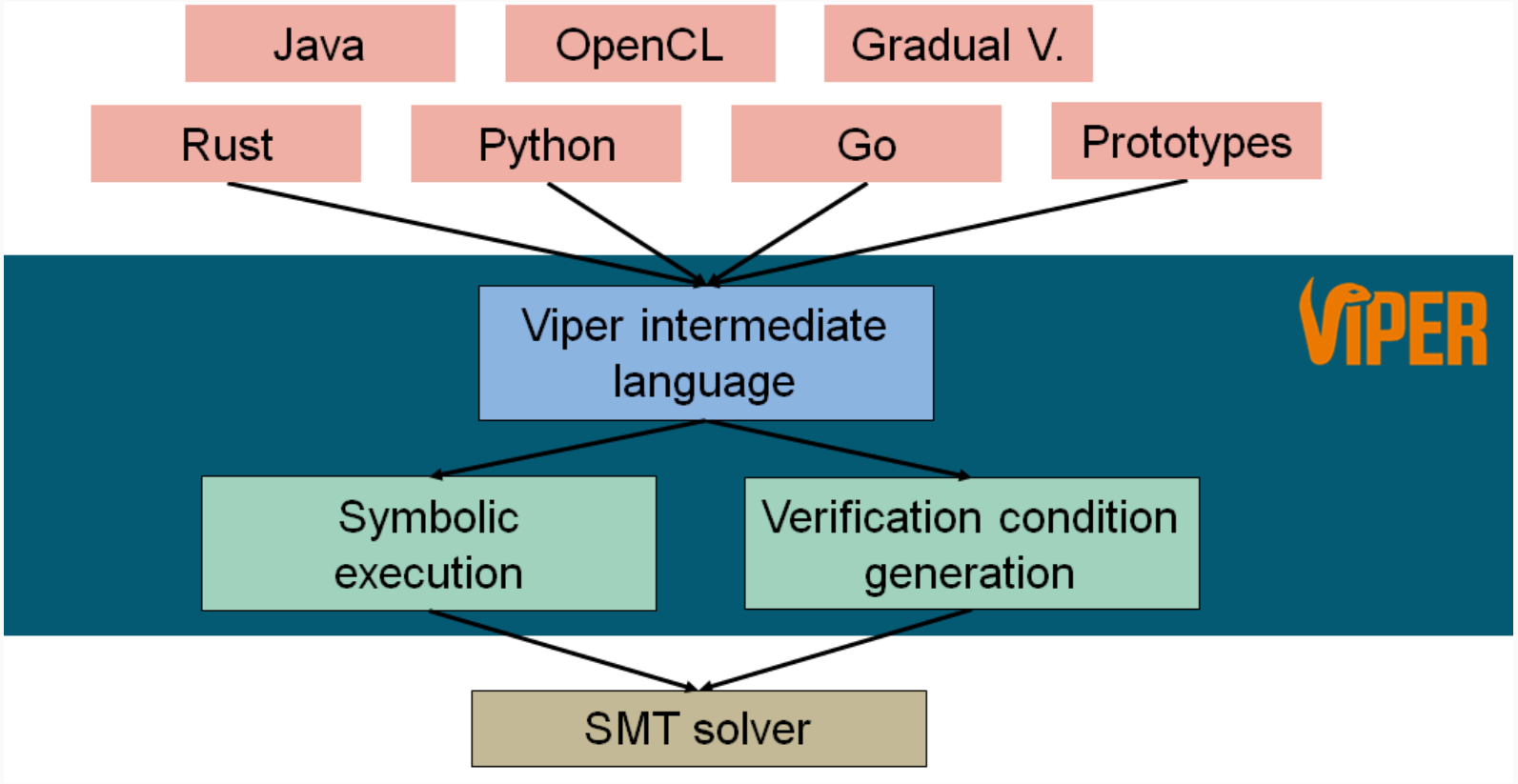


Empty footprint

Conclusion: the wand's syntax is not enough to infer its footprint.

A sound package algorithm for path conditions

An overview of Viper



From viper.ethz.ch (accessed on 24.07.2026)

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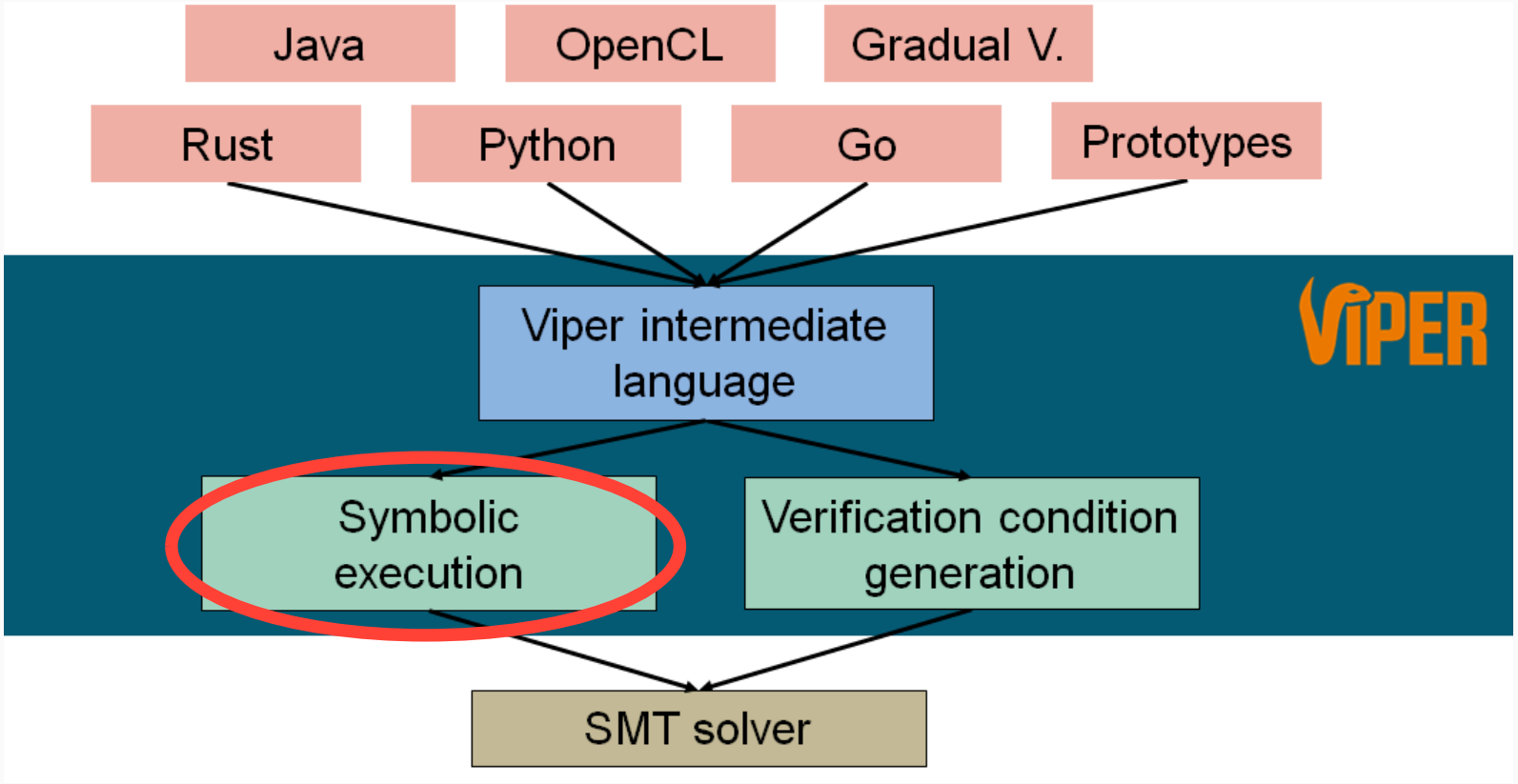

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Symbolic state

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Silicon: the symbolic execution backend of Viper

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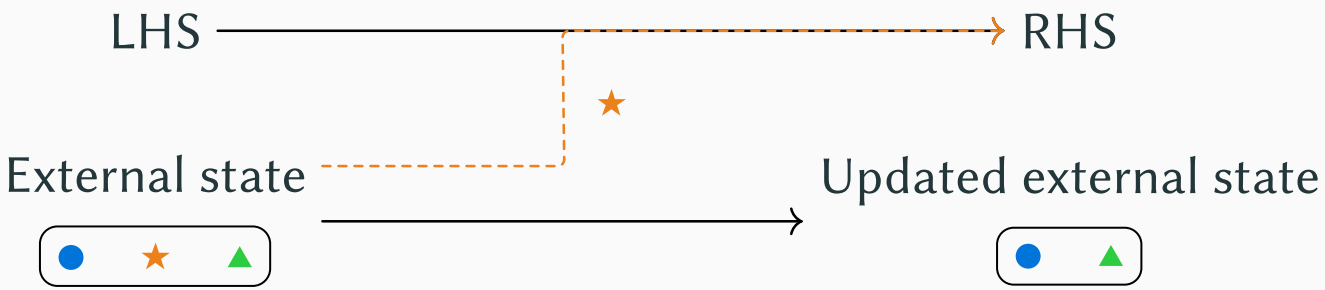
- `produce(a: Assertion, t: Term, s: State)(Q: State => ...)`
  adds the resources in a and assumes a holds

The existing package algorithm

Packaging a wand



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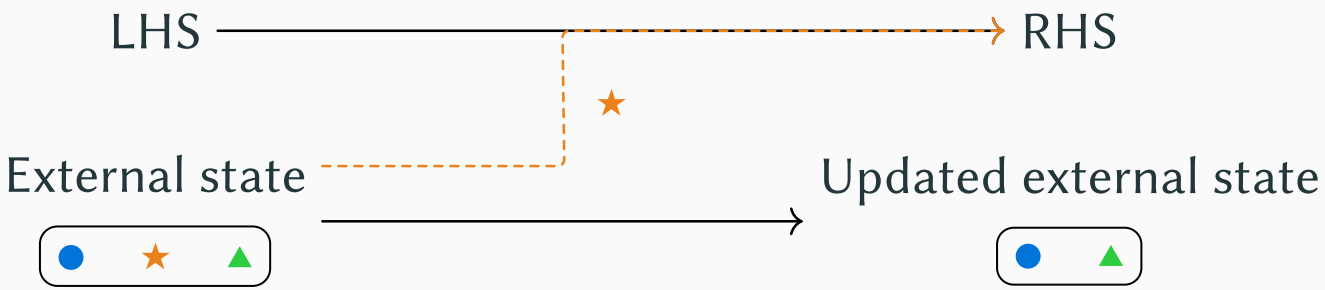

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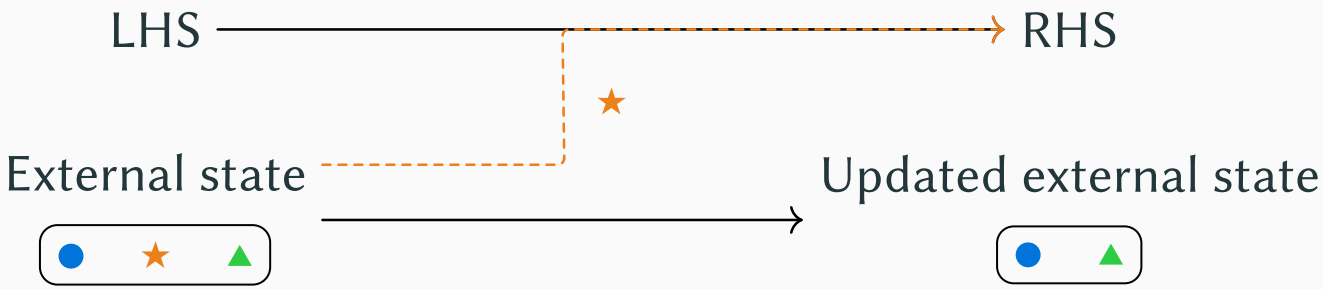

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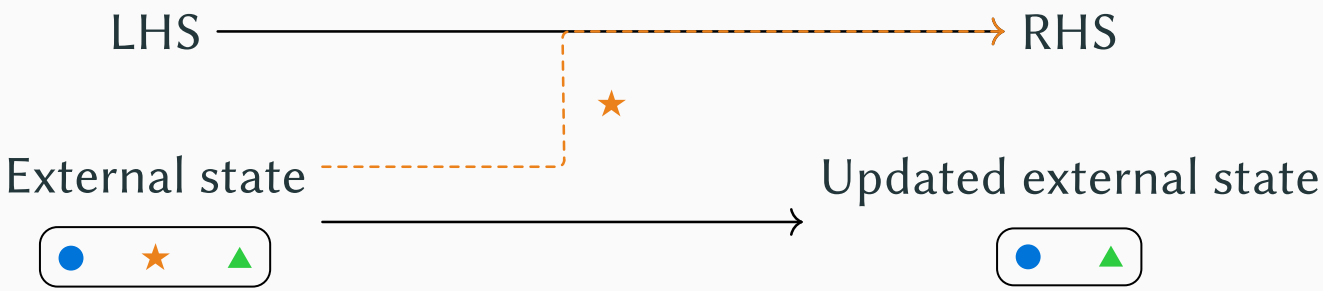

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Intuition: unsound to assume the branch at package time

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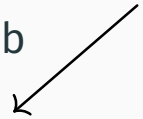

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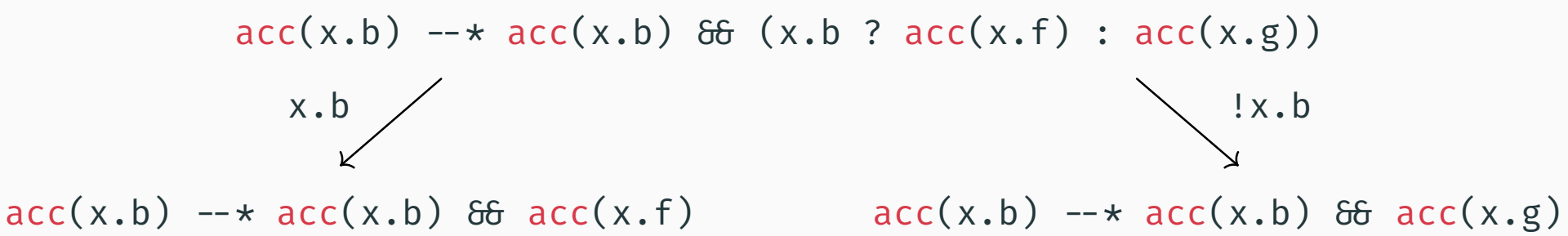

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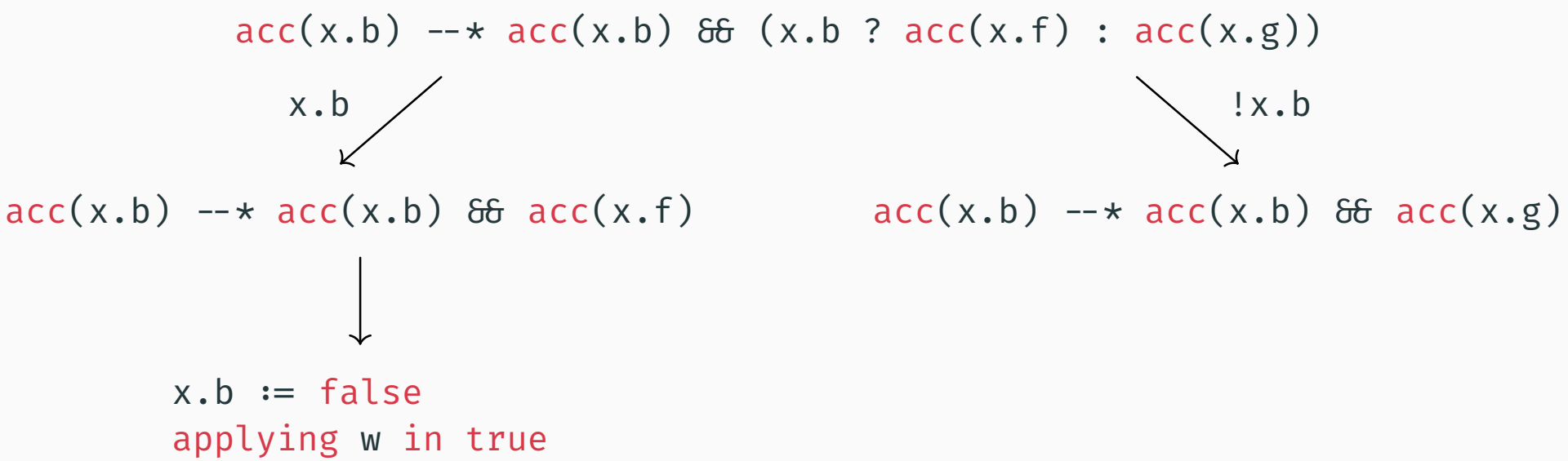

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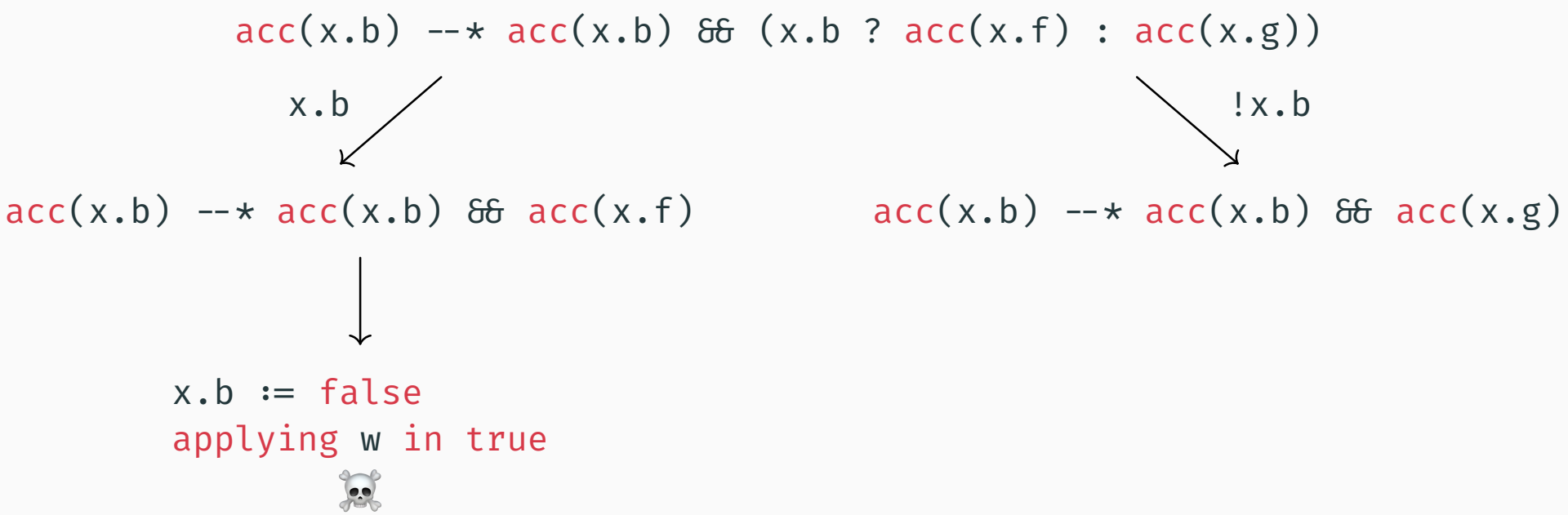

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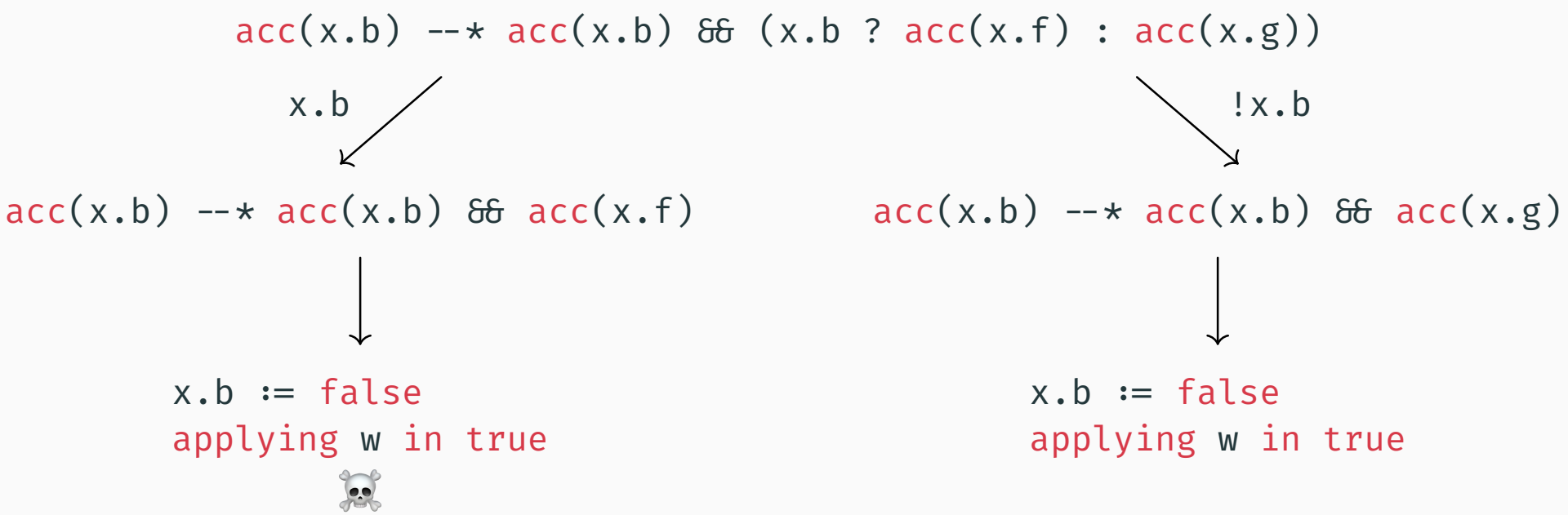

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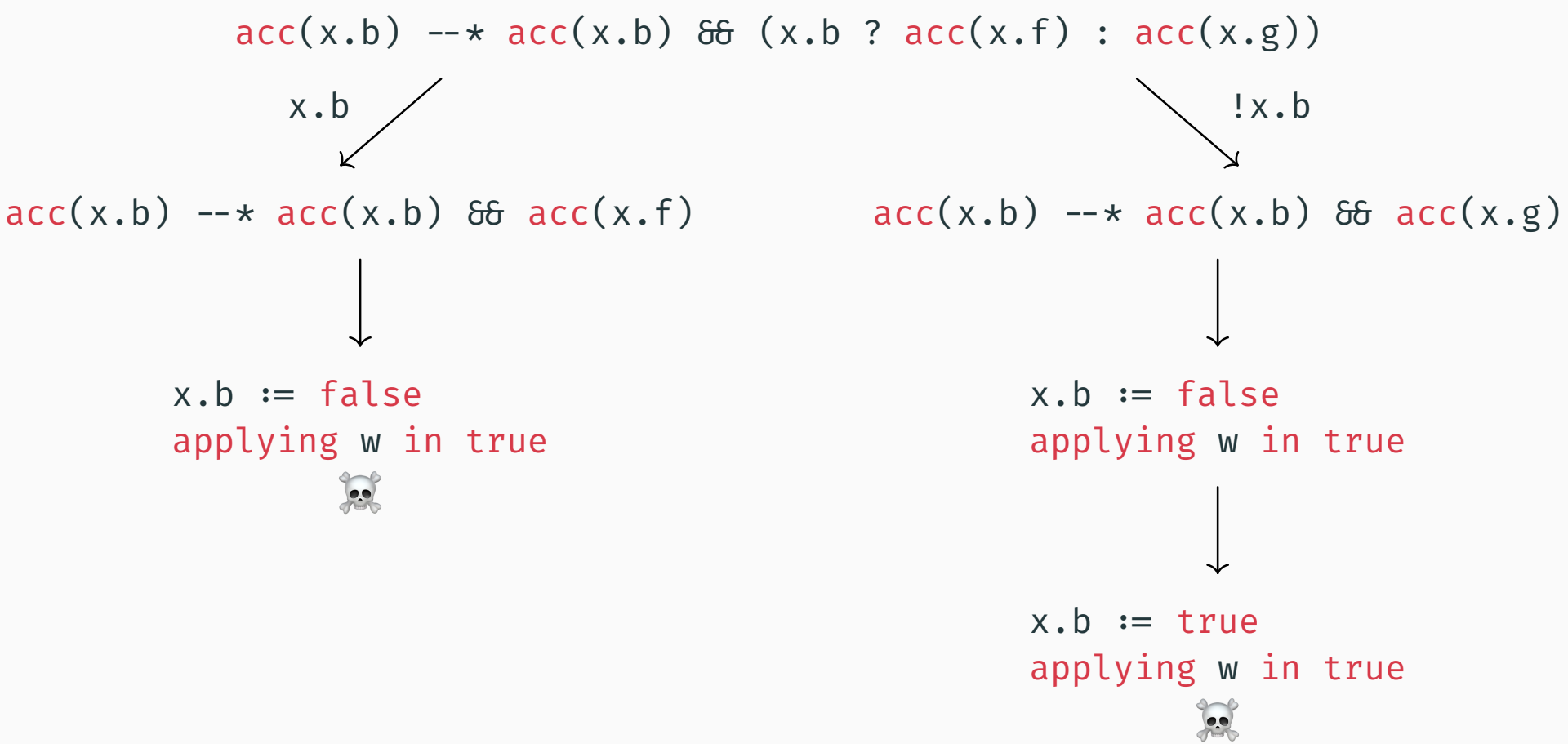

Unsound: `assert false` succeeds in both branches!

A sound packaging algorithm

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Yield a token $\mathtt{MW\_token}(\mathtt{w}, \mathtt{t}_{\mathtt{LHS}})$ when applying `w` with concrete LHS $\mathtt{t}_{\mathtt{LHS}}$

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Against the test suite

178 test files with wands (all passing with the previous encoding):

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- + 2 new test files (for the soundness issue described and another one)

Benchmark

On large verified codebases (Internet router [1] / verification library for security protocols [2])

Equivalent performance (details)

Modular functional specifications for wands

Attached facts by example (1)

The rationale: “attach” functional spec on top of an existing wand carrying permissions

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Classic wand

```
    list(cur)
    && len(cur) == len_cur
    && elems_positive(cur)
--* list(root)
    && len(root) == old(len(root))
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Wand and attached facts

```
list(cur) --* list(root)

attached old[lhs](len(cur)) == len_cur
          ⟹ len(root) == old(len(root))
     to list(cur) --* list(root)

attached old[lhs](elems_positive(cur))
          ⟹ elems_positive(root)
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```

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```

- **incremental** specification: first memory safety using the wands
- **modular** functional spec: each functional property specified and verified independently

Attached facts by example (2)

Attached facts come for free

```
while ( ... )
  invariant w
  invariant attached F to w
  invariant attached G to W
{
   ...
  package w {  ...  }
}
```

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Weakening properties

```
method weaken(x: Ref)
  requires true --* acc(x.f)
  requires attached x.f ⩾ 42 to true --* acc(x.f)
  ensures  true --* acc(x.f)
  ensures  attached x.f ⩾ 0  to true --* acc(x.f)
{ /* We don't need to do anything to prove the
postcondition. */ }
```

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Modular spec for wands

```
function f( ... ):  ...
  requires A && P1l --* B && P1r


method m( ... ) {
  package A && P1l && P2l --* B && P1r && P2r
  f( ... ) // error: wand not found
}
```

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  package A && P1l && P2l --* B && P1r && P2r
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}
```

```
function f( ... ):  ...
  requires A --* B
  requires attached old[lhs](P1l) ⟹ P1r to A --* B


method m( ... ) {
  package A --* B
  f( ... ) // works fine
}
```

Semantics of attached facts

Context: `attached e to A --* B`

- `e` is a (pure) boolean expression (no `acc( ... )`, predicates, or wands)

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$$[\![\texttt{attached e to w}]\!] \stackrel{\text{def}}{=} \forall t_A : \texttt{MW\_token}(\texttt{w}, t_A) \Rightarrow [\![\texttt{e}]\!]$$

- does not depend on t_A
- $\texttt{MW\_token}(\texttt{w}, t_A)$ allows accessing the wand definition

Evaluation of attached facts

On examples

Successfull rewrites of examples from papers [3], [4] + new ones (*e.g.* `delete_last.vpr`, `stalin-sort.vpr`)

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Early experiments with Prusti [5], [6]

Very promising on a few examples (*e.g.* binary search tree implem), wands naturally appear in Rust

```
struct Pair { fst: i32, snd: i32 }

fn borrow_first(p: &mut Pair) → &mut i32 {
  &mut p.fst
}
```

```
predicate pair(x: Ref) { acc(x.fst) && acc(x.snd) }

method borrow_first(p: Ref)
  requires pair(p)
  ensures  acc(p.fst)
  ensures  acc(p.fst) --* pair(p)
```

Accessing the footprint using attached expressions

Why is that different?

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Examples

Accessing the second element of a pair:

```
x.snd := 0
package acc(x.fst) --* pair(x) {  ...  }
var v: Int := attachedexp unfolding pair(x) in x.snd to W
assert v == 0
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(Computed) length of the prefix list:

```
while (...)
  invariant list(cur) --* list(root)
  invariant ...
  invariant (attachedexp len(root) - old[lhs](len(cur)) to W) == i
{ ... }
```

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(Computed) length of the prefix list:

```
while ( ... )
  invariant list(cur) --* list(root)
  invariant attachedexp_valid len(root) - old[lhs](len(cur)) to W
  invariant      (attachedexp len(root) - old[lhs](len(cur)) to W) == i
{ ... }
```

Validity conditions and semantics of attached expressions

Semantics

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$$\forall \texttt{t}_1, \texttt{t}_2 : \texttt{MW\_token}(\texttt{w}, \texttt{t}_1) \wedge \texttt{MW\_token}(\texttt{w}, \texttt{t}_2) \Rightarrow [\![\texttt{e}]\!][\texttt{t}_\texttt{A} := \texttt{t}_1] = [\![\texttt{e}]\!][\texttt{t}_\texttt{A} := \texttt{t}_2]$$

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- **not sufficient**: need a token to use that property: check the wand is not vacuously true

Appliability

Recall $\sigma \vDash A \mathrel{-\!\!*} B \iff \forall \sigma_A : \sigma_A \perp \sigma \land \sigma_A \vDash A \Rightarrow \sigma_A \uplus \sigma \vDash B$

A wand is *appliable* if there exists a valid LHS: $\exists \sigma_A : \sigma_A \perp \sigma \land \sigma_A \vDash A$

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Automation/integration in Silicon = future work

Conclusion

A sound algorithm and encoding for packaging path conditions

- introducing **magic wand tokens**

Attached facts

- enable **incremental** and **modular** functional specification of magic wands
- highly **automated** (come for free when packaging)
- promising application to Rust in Prusti

Attached expressions

- design space exploration
- **sufficient validity conditions** ✅ independence from the LHS ⚙️ appliability

Future work

Application to Rust Automation of appliability Formalization of this work

All artifacts available on github `remigerme/silver` and `remigerme/silicon` (see branches)

Example of joining heaps

Branch condition	Remaining heap	Conditionalized heap	Final joined heap
x.b - First(mwsf(t_A))	(x.g → ... # 1)	(x.g → ... # x.b ? 1 : 0)	(x.g → ... # x.b ? 1 : 0,
!x.b - !First(mwsf(t_A))	(x.f → ... # 1)	(x.f → ... # !x.b ? 1 : 0)	x.f → ... # !x.b ? 1 : 0)

Example of joining heaps

Branch condition	Remaining heap	Conditionalized heap	Final joined heap
`x.b` - `First(mwsf(`t_A`))`	`(x.g → ... # 1)`	`(x.g → ... # x.b ? 1 : 0)`	`(x.g → ... # x.b ? 1 : 0,`
`!x.b` - `!First(mwsf(`t_A`))`	`(x.f → ... # 1)`	`(x.f → ... # !x.b ? 1 : 0)`	`x.f → ... # !x.b ? 1 : 0)`

Wait, t_A is dangling... fine: this prevents the solver from accessing `x.f` and `x.g`.

Details of the tests

Failing test file	Comment
`conditionals.vpr` `SnapshotsBranching.vpr`	should have been failing in the first place as they exhibit a wrong footprint computation for wands (essentially described in [7])
`conditionals3.vpr` `PackageStateConsolidator.vpr`	can be successfully verified by enabling the SE-PC flag
`packaging_1.vpr`	true completeness regression

`conditionals.vpr` and `SnapshotsBranching.vpr`

```
method branching(x: Ref)
  requires acc(x.f) && acc(x.g, write) &&
acc(x.h, write)
{
  define A acc(x.f)
  define B acc(x.f) && (x.f ? acc(x.g) :
acc(x.h))

  package A --* B
  apply A --* B

  assert acc(x.f) && acc(x.g) && acc(x.h)
}
```

`packaging_1.vpr`

```
method incomplete(x)
  requires acc(x.f, 1/2)
{
  package acc(x.f, 1/4) && x.f == 2 --* acc(x.f,
1/4) && x.f == 2 {
      package true --* acc(x.f, 1/2) && x.f == 2
  }
}
```

Go back

Benchmarking the new encoding

Benchmarking on [1], [2] using hyperfine [8][1]:

Encoding	Mean [s]	Min [s]	Max [s]	Relative
old	139.253 ± 21.251	123.357	173.944	1.05 ± 0.17
new	132.219 ± 7.593	124.566	143.658	1.00

Table 1: Hyperfine benchmark results for the Internet router (5 runs per encoding).

Encoding	Mean [s]	Min [s]	Max [s]	Relative
old	25.433 ± 0.471	24.367	26.227	1.00
new	26.129 ± 0.647	25.138	27.397	1.03 ± 0.03

Table 2: Hyperfine benchmark results for the security protocol verification library (15 runs per encoding).

Go back

[1] All source files and the benchmark script can be found on my repository https://github.com/remigerme/silicon/blob/9ca9b342a9ad12dc888b573d7ca046ba9f749783/remi-demo

Generic evaluation procedure

Context: `attached e to A --* B`. Setup to evaluate e as in `A --* B && e`:

1. produce artificial LHS t_A
2. produce artificial RHS $mwsf(t_A)$
3. eval e in the resulting state: **the resulting term t_e depends on t_A**

```
def evalAttachedGeneric(e: Exp, w: Wand, s: State,
                        evalExp: (State, MWSF, Var, Term) => Term,
                        Q: (State, Term) => ...) => ...:
  consume(w, s)((mws, sW) =>
    tA = freshTerm()
    produce(w.lhs, tA, sW)(sA =>
      produce(w.rhs, mws.apply(tA), sA)(sB =>
        eval(e, sB)(tE =>
          tRes = evalExp(sB, mws, tA, tE)
          Q(sE, tRes)
        )
      )
    )
  )
```

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